Renormalization group in stochastic theory of developed turbulence 3

Operator-product expansion and scaling in the inertial range

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Outline

- Calculation of renormalization constants
- Homogeneous renormalization-group equation
- Invariant (running) parameters
- Infrared-stable fixed point
- Critical dimensions at the fixed point of RG
- Scaling in terms of physical variables
- Freezing of dimensions in the inertial range
- Operator-product expansion
- Renormalization of composite operators

Calculation of renormalization constants

The renormalization theorem states

$$G(\mathbf{J},\mathbf{J}') = \int \mathcal{D}v \int \mathcal{D}v' \, e^{S_{\mathrm{NS}}(\mathbf{v},\mathbf{v}')+\mathbf{vJ}+\mathbf{v}'\mathbf{J}'}$$

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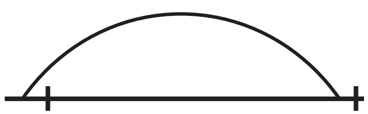
$$\Gamma_{Rvv'} = -\nu Z_\nu p^2 +$$  $$+ \ldots$$

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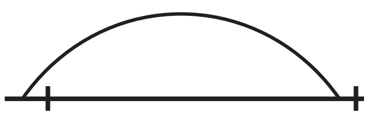
$$\Gamma_{Rvv'} = -\nu Z_\nu p^2 +$$  $$+ \ldots$$

Leading order $Z_\nu = 1$ in the graph, in the MS scheme then

$$Z_\nu = 1 - \frac{(d-1)\bar{S}_d}{8(d+2)} \frac{g_1}{\varepsilon} + \ldots \,, \qquad \bar{S}_d = 2\pi^{d/2}/(2\pi)^d \Gamma(d/2) \, .$$

Homogeneous renormalization-group equatio

Introduce a scaling parameter μ in the connection between renormalized and unrenormalized (bare) parameters:

$$\nu_0 = \nu Z_\nu , \qquad g_{01} = D_{01}\nu_0^{-3} = g_1 \mu^{2\epsilon} Z_\nu^{-3} \,.$$

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Bare quantities independent of μ: the homogeneous RG equation. For the pair correlation function, for instance,

$$\left[\mu\partial_\mu + \beta_1\partial_{g_1} - \gamma_\nu \nu \partial_\nu\right] G = 0 \, , \; \gamma_\nu = \mu\partial_\mu\big|_0 \ln Z_\nu \, , \; \beta_1 = \mu\partial_\mu\big|_0 g_1$$

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where derivatives taken with bare parameters fixed and

$$\int d\mathbf{r}\, \exp\left[\mathrm{i}(\mathbf{k}\cdot\mathbf{r})\right] \langle v_n(t,\mathbf{x}+\mathbf{r}) v_m(t,\mathbf{x})\rangle = P_{nm}(\mathbf{k}) G(k)$$

Invariant (running) parameters

RG solution for the velocity correlation function:

$$G(k) = \nu^2 k^{2-d} R\left(\frac{k}{\mu}, g_1, \frac{m}{\mu}\right) = \bar{\nu}^2 k^{2-d} R\left(1, \bar{g}_1, \frac{m}{k}\right) .$$

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Invariant (running) parameters $\bar{g}_1(\mu/k, g_1)$, $\bar{\nu}(\mu/k, g_1)$: first integrals of the RG equation, e.g.

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Connection between g_1 and ν yields the expression:

$$\beta_1(g_1, \varepsilon) = g_1 \left[-2\varepsilon + 3\gamma_\nu(g_1)\right] .$$

Infrared-stable fixed point

In one-loop approximation in the MS scheme

$$\gamma_\nu = (d-1)\bar{S}_d\, g_1/4(d+2),\ \ \beta_1 = g_1\left[-2\varepsilon + 3(d-1)\bar{S}_d\, g_1/4(d+2)\right].$$

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From the solution of the RG equation for $\bar{g}_1$ it follows that there is a physical ($g_1 > 0$) IR-attractive fixed point for $\varepsilon > 0$:

$$g_{1*} = 8(d+2)\varepsilon\,/\,3(d-1)\bar{S}_d, \quad \beta(g_{1*}) = 0, \quad \beta'(g_{1*}) = 2\varepsilon > 0\,.$$

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The value of γ_ν at the fixed point is found exactly:

$$\gamma_\nu^* \equiv \gamma_\nu(g_*) = 2\varepsilon/3,$$

without corrections of order ε^2, ε^3 etc.

Critical dimensions at the fixed point of RG

For $\varepsilon > 0$ $\exists$ an IR-stable fixed point: $\bar{g}_1 \to g_{1*} \propto \varepsilon$.
Asymptotics of functions W near this fixed point are generalized homogeneous functions

$$W\big|_{IR}(\{\lambda^{-\Delta_\omega} t_i\}, \{\lambda^{-1}\mathbf{x}_i\}) = \lambda^{\sum_\Phi \Delta_\Phi} W\big|_{IR}(\{t_i\}, \{\mathbf{x}_i\}) \,.$$

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They are all expressed through $\gamma_\nu^* \equiv \gamma_\nu(g_*)$:

$$\Delta_v = 1 - \gamma_\nu^*\,, \quad \Delta_{v'} = d - \Delta_\varphi\,, \quad \Delta_\omega = 2 - \gamma_\nu^*\,.$$

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Scaling for Green functions with separated arguments!

Scaling in terms of physical variables

IR fixed point yields large-scale limit ($k \to 0$, $u = m/k =$const)

$$G(k) \sim (D_{10}/g_{1*})^{2/3}\, k^{\,2-d-4\varepsilon/3} R(1, g_{1*}, u)\,,\ R(1, g_{1*}, u) = \sum_{n=1}^{\infty} \varepsilon^n R_n(u)$$

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Translate into traditional variables; trade D_{10} for the mean energy injection rate $\overline{\mathcal{E}}$ ($2 > \varepsilon > 0$):

$$\overline{\mathcal{E}} = \frac{(d-1)}{2(2\pi)^d} \int d\mathbf{k}\, d_f(k) \;\Rightarrow\; D_{10} = \frac{4(2-\varepsilon)\,\Lambda^{2\varepsilon-4}\overline{\mathcal{E}}}{\overline{S}_d(d-1)}\,,\; \Lambda = (\overline{\mathcal{E}}/\nu_0^3)^{1/4}\,.$$

Freezing of dimensions in the inertial range

Large-scale scaling in terms of $\overline{\mathcal{E}}$ and ν_0 for $2 > \varepsilon > 0$:

$$G(k) \sim \left[4(2-\varepsilon)/\overline{S}_d(d-1)g_{1*}\right]^{2/3} \nu_0^{2-\varepsilon}\,\overline{\mathcal{E}}^{\,\varepsilon/3} k^{\,2-d-4\varepsilon/3} R(1, g_{1*}, u)\,.$$

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Analysis of the inertial-range limit $u = m/k \to 0$ beyond RG.

Operator-product expansion

The limit $u = m/k \to 0$ beyond RG. To collect terms $\varepsilon \ln u \sim 1$, use *operator-product expansion* for composite operators F:

$$F_1(t, \mathbf{x}_1) F_2(t, \mathbf{x}_2) = \sum_\alpha C_\alpha(\mathbf{x}_1 - \mathbf{x}_2) F_\alpha \left[(\mathbf{x}_1 + \mathbf{x}_2)/2, t\right] .$$

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Statistical description of the turbulent flow by *structure functions* of the velocity field

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Anomalous scaling: exponents of S_n nonlinear in n.

Renormalization of composite operators

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Dangerous operators not known for $0 < \varepsilon < 2$: $u \to 0$ *safe!*